PROPOSAL ABSTRACT:

Name of Principal Investigator:	Christian Sadel
Proposal Title:	Linear and non-linear random operators on graphs

The main goal of the project is to enhance the current understanding of spectral theory for operators on graphs, to possibly construct more examples of Anderson models for which existence of absolutely continuous spectrum can be shown and finally to possibly construct certain graphs for which one can show GOE statistics of the eigenvalues of the Anderson model in a specific energy region. The projects of this research proposal are mostly based on my recent preprints [Sa4] and [SV].

In [Sa4] the Anderson model on antitrees of d-dimensional growth-rates for any $d > 1$ were considered. Special cases for integer d correspond to random operators of the form $P\Delta P + \mathcal{V}$ on $\mathbb{Z}^d$ where $\mathcal{V}$ is the random potential, Δ the usual Laplacian and P the radial orthogonal projection. These operators have a special form and are an example of what we called *operators with one propagating channel*. The astonishing finding was, that in a certain energy region, this model has a transition in the spectral type from pure point to absolutely continuous spectrum at the correct critical 2-dimensional growth rate. It has always been suspected that for graphs of more than the 2 dimensional growth with enough connections (edges) there should be absolutely continuous spectrum for the Anderson model, but the most interesting $\mathbb{Z}^d$ case seems to be far out of reach for current methods. This is the first example of some family of background graph operators with a finite dimensional (i.e. polynomial) growth rate where one can show existence of a.c. spectrum after adding an independent identically distributed potential. Previously this was only shown on infinite dimension trees and tree-like structures. This makes these models interesting and relevant.

In the proposed project we want to investigate the spectrum for these models in the energy regions that were not considered in [Sa4], moreover we intend to study non linear Schrödinger operators modeling effectively many particle systems. We also intend to generalize theorems from [Sa4] about operators with one propagating to some slightly more general framework and possibly apply it to certain random operators on constructed graphs within this framework. We also intend to study density of states and eigenvalue statistics for these models.

In [SV] we extended some results of [VV] showing existence of SDE limits for transfer matrix processes and GOE limiting statistics along special sequences of Anderson models along long boxes with rescaled random potential. The rescaling and thus decrease in variance of the random potential is imposed. One point of the work [Sa4] was that this rescaling happens somewhat through the graph structure, locally averaging an increasing number of random potentials along the shells and therefore effectively giving a decay in variance. Through this construction and combined with [VV] or [SV] one may be able to construct a graph for which the Anderson model truncated to growing finite pieces shows GOE statistics (convergence to sine_1 kernel). In terms of the so called 'universality conjectures' this would be an extremely interesting result.

BIBLIOGRAPHIC REFERENCES:

References

[Aiz] M. Aizenman, *Localization at weak disorder: some elementary bounds*, Rev. Math. Phys. **6**, 1163-1182 (1994)

[ASW] M. Aizenman, R. Sims and S. Warzel, *Stability of the absolutely continuous spectrum of random Schrödinger operators on tree graphs*, Prob. Theor. Rel. Fields, **136**, 363-394 (2006)

[AM] M. Aizenman and S. Molchanov, *Localization at large disorder and extreme energies: an elementary derivation*, Commun. Math. Phys. **157**, 245-278 (1993)

[AW] M. Aizenman and S. Warzel, *Resonant delocalization for random Schrdinger operators on tree graphs*, preprint J. Eur. Math. Soc., **15** (4), 1167-1222 (2013)

[AW2] M. Aizenman and S. Warzel, *Absolutely continuous spectrum implies ballistic transport for quantum particles in a random potential on tree graphs*, J. Math. Phys. **53**, 095205 (2012)

[AShW] M. Aizenman, M. Shamis and S. Warzel, *Resonances and Partial Delocalization on the Complete Graph*, Annales Henri Poincaré, published online, DOI: 10.1007/s00023-014-0366-9 (2014)

[BBR] S. Bachmann, M. Butz, W. de Roeck, *Disordered Quantum Wires: Microscopic Origins of the DMPK Theory and Ohm's Law*, J. Stat. Phys. **148**, 164189 (2012)

[Bou] J. Bourgain, *Random lattice Schrödinger operators with decaying potential: some higher dimensional phenomena.* Lecture Notes in Mathematics, **1807**, 70-99 (2003)

[BK] J. Bourgain and C. E. Kenig, *On localization in the continuous Anderson-Bernoulli model in higher dimension*, Inv. Math., **161**, 389-426 (2006)

[Br] J. Breuer, *Localization for the Anderson model on trees with finite dimensions*, Ann. Henri Poincarè **8**, 1507-1520 (2007)

[CKM] R. Carmona, A. Klein and F. Martinelli, *Anderson localization for Bernoulli and other singular potentials*, Commun. Math. Phys. **108**, 41-66 (1987)

[DLS] F. Delyon, Y. Levy and B. Souillard, *Anderson localization for multidimensional systems at large disorder or low energy*, Commun. Math. Phys. **100**, 463-470 (1985)

[DK] H. von Dreifus and A. Klein, *A new proof of localization in the Anderson tight binding model*, Commun. Math. Phys. **124**, 285-299 (1989)

[FHH] R. Froese, F. Halasan and D. Hasler, *Absolutely continuous spectrum for the Anderson model on a product of a tree with a finite graph*, J. Funct. Analysis **262**, 1011-1042

[FHS2] R. Froese, D. Hasler and W. Spitzer, *Absolutely continuous spectrum for the Anderson Model on a tree: A geometric proof of Klein's Theorem*, Commun. Math. Phys. **269**, 239-257 (2007)

[FHS3] R. Froese, D. Hasler and W. Spitzer, *Absolutely continuous spectrum for a random potential on a tree with strong transverse correlations and large weighted loops*, Rev. Math. Phys. **21**, 709-733 (2009)

[FKS] S. Fishman, Y. Krivolapov, A. Soffer, *The Nonlinear Schrdinger Equation with a random potential: Results and Puzzles*, Nonlinearity **25**, R53R72 (2012)

[FLSSS] R. Froese, D. Lee, C. Sadel, W. Spitzer and G. Stolz, *Localization for transversally periodic random potentials on binary trees*, preprint arXiv:1408.3961

[FMSS] J. Fröhlich, F. Martinelli, E. Scoppola and T. Spencer *Constructive proof of localization in the Anderson tight binding model*, Commun. Math. Phys. **101**, 21-46 (1985)

[FS] J. Fröhlich and T. Spencer, *Absence of diffusion in the Anderson tight binding model for large disorder or low energy*, Commun. Math. Phys. **88**, 151-184 (1983)

[GP] D. Gilbert and D. Pearson, *On subordinacy and analysis of the spectrum of one-dimensional Schrödinger operators*, J. Math. Anal. **128**, 30-56 (1987)

[GMP] Ya. Gol'dsheid, S. Molchanov and L. Pastur, *Pure point spectrum of stochastic one dimensional Schrödinger operators*, Funct. Anal. Appl. **11**, 1-10 (1977)

[Hal] F. Halasan, *Absolutely continuous spectrum for the Anderson model on trees*, PhD thesis 2009, accessible at arXiv:0810.2516v3 (2008)

[JL1] S. Jitomirskaya, Y. Last, *Power Law Subordinacy and Singular Spectra I. Half-line Operators*, Acta Math. **183**, 171-189 (1999)

[JL2] S. Jitomirskaya, Y. Last, *Power Law Subordinacy and Singular Spectra II. Line Operators*, Commun. Math. Phys. **211**, 643-658 (2000)

[KP] S. Kahn and D. Pearson, *Subordinacy and spectral theory for infinite matrices*, Helv. Phys. Acta **65**, 505-527 (1992)

[KLW] M. Keller, D. Lenz and S. Warzel, *On the spectral theory of trees with finite cone type*, Israel J. Math. **194**, 107-135 (2013)

[KLW2] M. Keller, D. Lenz and S. Warzel, *Absolutely continuous spectrum for random operators on trees of finite cone type*, J. D' Analyse Math. **118**, 363-396 (2012)

[KiLS] A. Kiselev, Y. Last and B. Simon *Modified Prüfer and EFGP transforms and the spectral analysis of one-dimensional Schrödinger operators*, Comm. Math. Phys. **194**, 1-45 (1997)

[Kl1] A. Klein, *Localization in the Anderson model with long range hopping*, Braz. J. Phys. **23**, 363-371 (1993)

[Kl2] A. Klein, *Spreading of wave packets in the Anderson model on the Bethe lattice*, Commun. Math. Phys. **177**, 755–773 (1996)

[Kl3] A. Klein, *Extended states in the Anderson model on the Bethe lattice*, Advances in Math. **133**, 163-184 (1998)

[KlLS] A. Klein, J. Lacroix and A. Speis, *Localization for the Anderson model on a strip with singular potentials*, J. Funct. Anal. **94**, 135-155 (1990)

[KS] A. Klein and C. Sadel, *Absolutely Continuous Spectrum for Random Schrödinger Operators on the Bethe Strip*, Math. Nachr. **285**, 5-26 (2012)

[KS2] A. Klein and C. Sadel, *Ballistic Behavior for Random Schrödinger Operators on the Bethe Strip*, J. Spectr. Theory **1**, 409-442 (2011)

[Klo] F. Klopp, *Weak disorder localization and Lifshitz tails*, Commun. Math. Phys. **232**, 125-155 (2002)

[KVV] E. Kritchevski, B. Valko and B. Virag, *The scaling limit of the critical one-dimensional random Schrodinger operator*, Commun. Math. Phys. **314** (2012), 775-806

[KuS] H. Kunz and B. Souillard, *Sur le spectre des operateurs aux differences finies aleatoires*, Commun. Math. Phys. **78**, 201-246 (1980)

[Lac] J. Lacroix, *Localisation pour l'opérateur de Schrödinger aléatoire dans un ruban*, Ann. Inst. H. Poincaré ser **A40**, 97-116 (1984)

[RS] R. A. Römer, H. Schulz-Baldes, *The random phase property and the Lyapunov spectrum for disordered multi-channel systems*, J. Stat. Phys. **140**, 122-153 (2010).

[Sa1] C. Sadel, *Relations between Transfer and Scattering matrices in the presence of hyperbolic channels*, J. Math. Phys. **52**, 123511 (2011)

[Sa2] C. Sadel, *Absolutely continuous spectrum for random Schrödinger operators on tree-strips of finite cone type*, Annales Henri Poincaré, **14**, 737-773 (2013)

[Sa3] C. Sadel, *Absolutely continuous spectrum for random Schrödinger operators on the Fibbonacci and similar tree-strips*, Math. Phys. Anal. Geom., published online, DOI 10.1007/s11040-014-9163-4

[Sa4] C. Sadel, *Anderson transition at 2 dimensional volume growth on antitrees and spectral theory for operators with one propagating channel*, preprint, arXiv:1501.04287v2

[SS1] C. Sadel, H. Schulz-Baldes, *Scaling diagram for the localization length at a band edge*, Annales Henri Poincare **8**, 1595-1621 (2007).

[SS3] C. Sadel, H. Schulz-Baldes, *Random Dirac operators with time-reversal symmetry*, Comm. Math. Phys., **295**, 209-242 (2010)

[SS4] C. Sadel, H. Schulz-Baldes, *Random Lie group actions on compact manifolds: a perturbative analysis*, Anals of Prob. **38**, No 6, 2224-2257 (2010)

[SV] C. Sadel, B. Virag, *A central limit theorem for products of random matrices and GOE statistics for the Anderson model on long boxes*, preprint, arXiv:1411.1040 (2014)

[Sha] M. Shamis, *Resonant delocalization on the Bethe strip*, Annales Henri Poincare, published online, DOI 10.1007/s00023-013-0280-6

[SB] H. Schulz-Baldes, *Rotation numbers of Jacobi matrices with matrix entries*, Math. Phys. Elect. Journal **13**, (2007).

[SW] B. Simon and T. Wolff, *Singular continuum spectrum under rank one perturbations and localization for random Hamiltonians*, Commun. Pure. Appl. Math. **39**, 75-90 (1986)

[Sto] G. Stolz, *An introduction to the mathematics of Anderson localization, Entropy and the quantum II*, 71108, Contemp. Math. 552, Amer. Math. Soc., Providence, RI, 2011.

[Tau] M. Tautenhahn, *Localization criteria for Anderson Models on locally finite graphs*, J. Stat. Phys. **144**, 6075 (2011)

[VV] B. Valko, B. Virag, *Random Schrödinger Operators on long boxes, noise explosion and the GOE*, Trans. Amer. Math. Soc. **366**, 37093728 (2014)

[Wa] W.-M Wang, *Localization and universality of Poisson statistics for the multidimensional Anderson model at weak disorder*, Invent. Math. **146**, 365-398 (2001)

[WZ] W.-M. Wang, Z. Zhang, *Long Time Anderson Localization for the Nonlinear Random Schrdinger Equation*, J. Stat. Phys. **134**, 953-968 (2009)