A Hardy-Lieb-Thirring inequality for fractional Pauli operators

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Abstract

In this talk we will discuss recent work on Hardy-Lieb-Thirring inequalities for the Pauli operator. The classical Lieb-Thirring inequality estimates the sum of the negative eigenvalues of a Schrödinger operator $-\Delta + V$ by an integral of a power of the potential. In 3-dimensions, this becomes

$$\mathbf{tr}(-\Delta + V)_{-} \leq C \int (V(x))_{-}^{5/2}\, dx$$

The classical Hardy inequality states that (also in 3D),

$$-\Delta - \frac{1}{4|x|^2} \geq 0,$$

where the constant $\frac{1}{4}$ is the sharp constant for this bound. It is well known, that these inequalities can be combined to yield "Hardy-Lieb-Thirring inequalities", i.e. the Lieb-Thirring inequality above still holds (possibly with a different constant) if V is replaced by $-\frac{1}{4|x|^2} + V$ on the left side.

In this talk we will discuss similar inequalities, where the non-relativistic kinetic energy operator $-\Delta$ is replaced by a magnetic Pauli operator. In particular, we will discuss a relativistic version, where the kinetic energy is the square root of a Pauli operator, and where $\frac{1}{4|x|^2}$ is replaced by $\frac{c_H}{|x|}$, with c_H being the critical Hardy constant for the relativistic problem.

This is joint work with Gonzalo Bley.