CORRIGENDUM TO “L^p-GENERIC COCYCLES HAVE ONE-POINT LYAPUNOV SPECTRUM”

ALEXANDER ARBIETO AND JAIRO BOCHI

Lemma 4 in [1] is not true, unless $k = 1$ (this case was used in page 78, line 9). For arbitrary k, the correct statement is:

Lemma 4'. *Given $k \in \{1, \dots, d\}$ and $P > 0$, there is $C = C(P, k, d) > 0$ such that for every A, $B \in \mathrm{GL}(d, \mathbb{R})$ with $\|A\|$, $\|B\| \le P$, we have*

$$\log^+ \|\wedge^k B\| \le \log^+ \|\wedge^k A\| + C\|B - A\|.$$

Proof. Let $L = \wedge^k B - \wedge^k A$. Let $\{e_i;\ 1 \le i \le d\}$ be an orthonormal basis of $\mathbb{R}^d$. Then $\{e_{i_1} \wedge \cdots \wedge e_{i_k};\ 1 \le i_1 < \cdots < i_k \le d\}$ is an orthonormal basis of $\wedge^k \mathbb{R}^d$. We have

$$\begin{aligned} L(e_{i_1} \wedge \cdots \wedge e_{i_k}) = {} & (B - A)e_{i_1} \wedge Be_{i_2} \wedge \cdots \wedge Be_{i_k} + \\ & Ae_{i_1} \wedge (B - A)e_{i_2} \wedge Be_{i_3} \wedge \cdots \wedge Be_{i_k} + \cdots + \\ & Ae_{i_1} \wedge \cdots \wedge Ae_{i_{k-1}} \wedge (B - A)e_{i_k}. \end{aligned}$$

It follows that $\|L(e_{i_1} \wedge \cdots \wedge e_{i_k})\| \le kP^{k-1}\|B - A\|$. Therefore $\|L\| \le C\|B - A\|$ for some constant C. Then $\|\wedge^k B\| \le \|\wedge^k A\| + C\|B - A\|$ and using that $\log^+(x + y) \le \log^+ x + y$ for all x, $y \ge 0$, the lemma follows. □

Now we indicate the necessary modifications in the proof of part (a) of Theorem 2. In the case condition (3) is satisfied, we let N and K be as before. Take $P = e^{KN}$ and let C be given by lemma 4'. We modify the definition of δ' in (5) to $\delta' = \min\{\eta, \varepsilon C^{-1} e^{-K(N-1)}\}$. If $x \in G$ then $\|A^N(x)\|$, $\|B^N(x)\| \le P$. So we can apply lemma 4' and the estimative (8) becomes

$$\frac{1}{N} \int_G \log^+ \|\wedge^k B^N\| \le \Lambda_k(A) + 2\varepsilon + \frac{C}{N} \int_G \|B^N - A^N\|.$$

Then the upper bound in (9) becomes $\Lambda_k(A) + 3\varepsilon$, and so the first case of part (a) follows.

For the general case, we replace the first part in (10) with $k \int_{L_a} \log^+ \|A\| < \varepsilon$. Then in page 80, line 2, we can further estimate

$$\cdots \le \int_{L_a} \log^+ \|\wedge^k B\| \le k \int_{L_a} \log^+ \|B\|.$$

Using lemma 4 with $k = 1$, we can change line 4 to

$$\int_{L_a} \hat{\lambda}_k(B, x)\, d\mu(x) \le k \int_{L_a} \log^+ \|A\| + k \int_{L_a} \|B - A\|.$$

Then we finish the proof in the same way as before.

Date: July 15, 2003.

References

[1] A. Arbieto and J. Bochi. L^p-generic cocycles have one-point Lyapunov spectrum. *Stochastics and Dynamics*, 3 (2003) no. 1, 73–81.

Alexander Arbieto (`alexande@impa.br`) Jairo Bochi (`bochi@impa.br`)

IMPA, Estrada D. Castorina 110, Jardim Botânico, 22460-320 Rio de Janeiro, Brazil