CORRECTION TO THE PAPER "OPENING GAPS IN THE SPECTRUM OF STRICTLY ERGODIC SCHRÖDINGER OPERATORS"

As noted by Zhiyuan Zhang, Lemma 8.1 in the paper [1] is incorrect. The error occurs in page 88: the family $\tilde{A}_t$ is not necessarily contained in the set $\mathcal{B}$. It is not difficult to correct the lemma by imposing the appropriate hypotheses on the set $\mathcal{B}$. But since the lemma plays an important role in the rest of the paper, a few corrections need to be made elsewhere as well.

We introduce some notation. Let $\mathbb{B}, \mathbb{B}' \subset \mathrm{SL}(2,\mathbb{R})$ be bounded open neighborhoods of the identity matrix; then so are the following sets:

$$\mathbb{B}\mathbb{B}' := \{BB' \colon B \in B,\ B' \in \mathbb{B}'\}, \qquad \mathbb{B}^{-1} := \{M^{-1} \colon M \in \mathbb{B}\}$$

Given a cocycle $A \in C(X, \mathrm{SL}(2,\mathbb{R}))$, the sets

$$\mathcal{N}(A,\mathbb{B}) := \left\{\tilde{A} \in C(X,\mathrm{SL}(2,\mathbb{R})) \colon A(x)^{-1}\tilde{A}(x) \in \mathbb{B} \text{ for all } x \in X\right\},$$
$$\mathcal{N}(\mathbb{B},A) := \left\{\tilde{A} \in C(X,\mathrm{SL}(2,\mathbb{R})) \colon \tilde{A}(x)A(x)^{-1} \in \mathbb{B} \text{ for all } x \in X\right\}$$

are open neighborhoods of A in $C(X, \mathrm{SL}(2,\mathbb{R}))$. If the neighborhood $\mathbb{B}$ is homeomorphic to $\mathbb{R}^3$ then we call it a *gauge*; in that case, the set $\mathcal{N}(A,\mathbb{B})$ (resp. $\mathcal{N}(\mathbb{B},A)$) is called the *basic right* (resp. *left*) *neighborhood* of A with gauge $\mathbb{B}$.

We modify the statements of Lemma 8.1, Proposition 9.1, and Proposition 9.2 by adding the assumption that the set $\mathcal{B}$ is a basic (either left or right) neighborhood. Contractibility becomes automatic and may be removed from the hypotheses. The other statements in the paper are unaffected. The proofs require minor modifications, as we describe below.

Notice if $\mathbb{B}$ is a gauge then so is $\mathbb{B}^{-1}$, and moreover $\tilde{A} \in \mathcal{N}(A,\mathbb{B})$ iff $\tilde{A}^{-1} \in \mathcal{N}(\mathbb{B}^{-1}, A^{-1})$. Therefore in order to prove either Lemma 8.1, Proposition 9.1, or Proposition 9.2, it is sufficient to consider basic right neighborhoods, because the corresponding statement for basic left neighborhoods follows by considering the inverse cocycles over the homeomorphism f^{-1}.

Let us review the proof of Lemma 8.1. Assume that $\mathcal{B} = \mathcal{N}(A_*,\mathbb{B})$ is a basic right neighborhood determined by a cocycle A_* and a gauge $\mathbb{B}$. Suppose the family of cocycles A_t, $t \in \partial[0,1]^p$, is contained in $\mathcal{B}$. By compactness, the family is also contained in $\mathcal{N}(A_*,\mathbb{B}')$ for some gauge $\mathbb{B}'$ whose closure is contained in $\mathbb{B}$. Taking $\varepsilon > 0$ small enough, the set $\mathbb{B}_\varepsilon := \{M \in \mathrm{SL}(2,\mathbb{R}) \colon \|M - \mathrm{Id}\| < \varepsilon\}$ is a gauge and moreover $\mathbb{B}'\mathbb{B}_\varepsilon \subset \mathbb{B}$. Then the rest of the proof of Lemma 8.1 goes as before and the resulting family indeed belongs to $\mathcal{B}$.

The sets $\mathcal{B}_\varepsilon(A)$ defined in page 89 should be replaced by basic neighborhoods of A, either right or left according to the case. It is convenient to restate Proposition 6.3 in terms of basic neighborhoods. Then the adaptations to the proof of Proposition 9.1 become straightforward.

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The proof of Proposition 9.2 needs to be rewritten, with appropriate gauges $\mathbb{B}_n$ and $\mathbb{G}_{q,n}$ playing the respective roles of the numbers ε_n and $r_n^{(q)}$. More precisely:

1. We fix a decreasing sequence of gauges $\mathbb{B}_1 \supset \mathbb{B}_2 \supset \cdots$ converging to $\{\mathrm{Id}\}$ such that if τ, $\tau' \in \partial[0,1]^p$ are 2^{-n}-close then $A_{\tau'} \in \mathcal{N}(A_\tau, \mathbb{B}_n)$.
2. As before, the unit cube is stratified into subcubes, shell etc. The major task is to extend continuously A_t to the shell while creating invariant sections. The definition is done inductively, starting with the vertices, then the edges, and so on.
3. For each vertex $t \in K_0$, define $\tau(t) \in \partial[0,1]^p$ and A_t as before, but now with $A_t \in \mathcal{N}(A_{\tau(t)}, \mathbb{B}_n)$.
4. Next consider an arbitrary edge F in the shell. Select a distinguished vertex t_0 of F, and let t_1 be the other vertex. Letting 2^{-n} be the length of the edge, we have, by analogous reasons as before,

$$A_{t_1} \in \mathcal{N}(A_{\tau(t_1)}, \mathbb{B}_{n-1}) \subset \mathcal{N}(A_{\tau(t_0)}, \mathbb{B}_{n-3}\mathbb{B}_{n-1}) \subset \mathcal{N}(A_{t_0}, \mathbb{B}_{n-1}^{-1}\mathbb{B}_{n-3}\mathbb{B}_{n-1})\,.$$

   For large enough n we can take a gauge $\mathbb{G}_{0,n}$ containing $\mathbb{B}_{n-1}^{-1}\mathbb{B}_{n-3}\mathbb{B}_{n-1}$. We apply Proposition 9.1 to define an extended family $A_t \in \mathcal{N}(A_{t_0}, \mathbb{G}_{0,n})$, $t \in F$ with an extended family of sections. The gauges $\mathbb{G}_{0,n}$ can be chosen tending to $\{\mathrm{Id}\}$ as $n \to \infty$.
5. Let $q \in \{1, 2, \ldots, p-1\}$. By induction, assume that the extensions were already made to all cells of dimension q in the shell, and that we have constructed a sequence of gauges $\mathbb{G}_{q-1,n}$, $n \geq n_0$, that tends to $\{\mathrm{Id}\}$ as $n \to \infty$, so that following property holds: If F is a q-dimensional cell of diameter 2^{-n} then F has a distinguished vertex t_0 such that $A_t \in \mathcal{N}(A_{t_0}, \mathbb{G}_{q-1,n})$ for all $t \in F$.
6. Now consider a cell F of dimension $q+1$; its relative boundary B is composed of 2^{q+1} cells of dimension q, each of which having a distinguished vertex. Select one of these vertices t_0 and call it the distinguished vertex of F. Let $\mathbb{G}_{q,n}$ be a gauge containing $(\mathbb{G}_{q,n}^{-1}\mathbb{G}_{q,n})^{2^{q+1}}$. Then $A_t \in \mathcal{N}(A_{t_0}, \mathbb{G}_{q,n})$ for all $t \in B$. We apply Proposition 9.1 to define an extended family $A_t \in \mathcal{N}(A_{t_0}, \mathbb{G}_{q,n})$, $t \in F$ with an extended family of sections. The gauges $\mathbb{G}_{q,n}$ can be chosen tending to $\{\mathrm{Id}\}$ as $n \to \infty$.
7. We have defined the extensions to the shell. Continuity on the boundary of the cube is easily checked, basically as before. To conclude the proof, Proposition 9.1 is applied once more to find the extensions from the shell to the whole cube.

It is also necessary to modify the proof of Lemma 10.1, as follows. In that part, let $\mathcal{B}_\varepsilon(A)$ denote the basic left neighborhood $\mathcal{N}(\mathbb{B}_\varepsilon, A)$, where $\mathbb{B}_\varepsilon := \{M \in \mathrm{SL}(2,\mathbb{R})\colon \|M - \mathrm{Id}\| < \varepsilon\}$. If δ_n is sufficiently small then the set $\mathcal{T}_n$ defined in page 91 is a basic left neighborhood of A. So the proof of Lemma 10.1 goes as before, using the left version of Proposition 9.2.

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References

[1] Avila, A.; Bochi, J.; Damanik, D. Opening gaps in the spectrum of strictly ergodic Schrödinger operators. *J. Eur. Math. Soc.* 14 (2012), no. 1, 61–106.