The volume of a cone

Let $D \subset \mathbb{R}^n$ be a set with $n$-volume $|D|$, and let $P = (a_1, \ldots, a_n, h) \in \mathbb{R}^{n+1}$ be a fixed point with $h > 0$. We consider that cone $C$ consisting of all line segments from $P$ to a point in $D$. The cone can be parametrized with the variables $x_1, \ldots, x_n, t$, where $(x_1, \ldots, x_n) \in D$, $t \in [0, 1]$ and the equations

$$(y_1, \ldots, y_{n+1}) = (x_1, \ldots, x_n, 0) + t(x_1 - a_1, \ldots, x_n - a_n, -h).$$

It is easy to see that

$$\left| \frac{\partial(y_1, \ldots, y_{n+1})}{\partial(x_1, \ldots, x_n, t)} \right| = ht^n;$$

and it follows that $n + 1$-volume of $C$ is

$$|C| = \frac{1}{n+1} |D|h.$$