Let $F : D \subset \mathbb{R}^2 \to \mathbb{R}^2$ be a $C^2$ mapping defined in $D$ given by $u = u(x, y), v = v(x, y)$. Let $R \subset D$ be a subregion with smooth boundary $\partial R \subset D$. Then

$$\iint_R (u_x v_y - u_y v_x) \, dx \, dy = \int_{\Gamma} u \, dv,$$

where $\Gamma = F(\partial R)$. The proof is based on integration by parts of the left hand side.