Volume of Revolution

- if $y = f(x)$ is a graph with respect to the line $L : y = mx + n$ then $1 + mf'(x) \neq 0$
- let $V$ be the volume generated by rotating the region $R$ bounded by the curve $y = f(x)$ as a graph over the line $L$
- relation between $\Delta s$ and $\Delta x$:

\[
\Delta s = \text{projection of the vector } (\Delta x, \Delta y) \text{ in the direction } (\cos \theta, \sin \theta)
\]

\[
= \left| \left(1, \frac{\Delta y}{\Delta x}\right) \cdot (\cos \theta, \sin \theta) \right| \Delta x
\]

\[
= \frac{1}{\sqrt{1 + m^2}} |1 + mf'(x)| \Delta x
\]

\[
\Rightarrow \quad V = \frac{\pi}{(1 + m^2)^{\frac{3}{2}}} \int_a^b (f(x) - mx - n)^2 |1 + mf'(x)| dx
\]