Preliminary: \( (D, T) \) graded category, \( F : D \to D' \) functor between graded categories is called graded if there exists a natural

isomorphism \( T_F : F \circ T \cong T \circ F \).

A natural transformation \( \eta \) between graded functors is called graded.

\[
\begin{array}{c}
F \circ T \\
\downarrow \eta_T \\
T \circ F \\
\downarrow \eta_F \\
G \circ T \\
\end{array}
\]

\( F : D \to D' \) functor is graded and exact if \( X \to Y \to Z \to TX \)

\( \Rightarrow FX \to FY \to FZ \to FTX = T(FX) \) is exact.

**Definition (Seme Functor):** \( D \) is linear category (\( k \) a field) with finite dimensional \( \text{Hom} \).

A covariant additive functor \( S : D \to D \) is called a

Seme functor.

\( S \) is an equivalence.

There are bi-functional maps \( \eta_{AB} : \text{Hom}_D(A, B) \to \text{Hom}_D(A, SB) \) ∀\( A, B \).

Properties:

1. Any autolequivalence \( \Phi : D \to D \) commutes with a Seme functor \( \Phi \circ S \cong S \circ \Phi \).
2. If \( D \) is Ded \( \Rightarrow S \) is exact.
3. Any two Seme functors are connected by a canonical functorial

isomorphism with commutes both \( \eta_{AB} \).

**Proof:** \( S, S' \) two Seme functors, \( A \in \text{ob}(D) \).

\[
\begin{align*}
\text{Hom}_D(A, A) & \cong \text{Hom}_D(A, S(A)) \\
\eta_A & \cong SA \\
\eta_A & \cong SA
\end{align*}
\]

\( \eta_A \cong SA \Rightarrow SA \cong SA \).
Reconstruction: \( X = \text{smooth, dg, scheme} \), \( n = \text{dim } X \), \( D = D^b(X) \)

\( \Omega_X = \text{canonical sheaf} \)

\( \Rightarrow \) The same functor is \( (\cdot) \otimes \Omega_X \otimes \cdot \) due to the same Grothendieck duality \( \text{Ext}^n(F, G) = \text{Ext}^{-n}(G, F \otimes \Omega_X)^* \).

\( \Rightarrow \) For a closed point, \( x \in X \), let \( k(x) \) be the residue field.

Notation: \( \text{Ann}^n(P, Q) := \text{Ext}^n(P, Q) = \text{Hom}(P, Q[n]) \)

Prop: A point object in \( D \) in codim \( n \) is an obj. \( P \) of \( D \) that satisfies

1. \( \text{SP} = P[n] \)
2. \( \text{Hom}^n(P, P) = 0 \)
3. \( \text{Hom}^n(P, P) = K(P) \text{ is a field extension of } k \).

Proof: (Identify as closed points) Say \( d \) is ample or antivalence. Then \( P \in D \text{coh}(X) \) is a point object (codim \( n \) is already pointed) \( \Leftrightarrow \)

\( P \cong \mathcal{O}_X[d] \), \( d \in \mathbb{Z} \) isomorphic to the skyscraper sheaf of a closed point \( x \in X \).

Proof: \( x \in X \text{ closed pt} \Rightarrow k(x) \text{ skyscraper sheaf} \Rightarrow k(x) \text{ is a pt in } D^b(X) \)

of codim \( n \).

1. \( k(x) \otimes \Omega_X \cong k(x) \cdot \Omega_X \)
2. \( \text{Hom}^n(k(x), k(x)) = 0 \)
3. \( \text{Hom}^n(k(x), k(x)) = k(x) \)

let \( P \) be a point object in \( D \) of codim \( n \). let \( H^* \) be cohomology sheaves of \( P \),

(i) \( \text{support } P = \text{finite points} \) (bounded above cot.)
Consider the spectral sequence: $E^{p,q}_2 = \bigoplus_{k+j=p} \text{Ext}^j_{\mathbb{P}^2}(M,k I) \Rightarrow \text{Hom}_{\mathbb{P}^2}(M^*, P)$

If $k-j$ is finite, then $k$ is supported on a single point $p$.

This follows from the fact that the space of $k$ is minus the space spanned by the generators of the first to the $k$th grade, and the second.

$M$ is the space spanned by the vanishing cycle of the minimal support z-module $M$.

Consider $\text{Hom}^0_{\mathbb{P}^2}(M^*, k I) \neq 0$, with minimal $k-j$. Then space spans $M$ too.

$\Rightarrow k-j = 0$. Therefore, all but one cohomology sheaves are trivial.