Geometric construction of minimizers to least gradient problems: the density case

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Abstract

Given a bounded domain $\Omega \subset \mathbb{R}^d$ for any dimension $d \geq 2$, we construct a continuous solution of the variational problem

$$\inf \left\{ \int_\Omega \mathfrak{a}(x)|Du| : \ u \in BV(\Omega), \ u\big|_{\partial\Omega} = g \right\},$$

where $g \in C(\partial\Omega)$, and $\mathfrak{a} \in C^2(\bar{\Omega})$ is a uniformly-positive density function. Under suitable geometric conditions on the domain Ω, we extend in [1] the Sternberg-Williams-Ziemer procedure [2], to this inhomogeneous setting. The level sets of the minimizer are minimal surfaces in the conformal metric $(\mathfrak{a}(x))^{2/(d-1)}\mathbb{I}_d$. This approach complements the one developed by Jerrard-Nachman-Moradifam [3], as it provides a continuous solution even in higher dimensions $d \geq 8$ where the level sets may develop singularities. The proof relies on an application of a strict maximum principle for sets with area-minimizing boundary, established by Leon Simon [4].

References

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