Regularity of deformations that minimise the neo-Hookean energy

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Abstract

Not every map u in the Sobolev space $W^{1,2}(\Omega;\mathbb{R}^3)$ represents a physically possible deformation of an elastic body $\Omega \subset \mathbb{R}^3$. Identifying a suitable subclass $\mathcal{A} \subset W^{1,2}(\Omega;\mathbb{R}^3)$ that models real deformations, subjected to some fixed Dirichlet boundary condition, and admits the existence of a minimizer of the neo-Hookean energy

$$E(u) = \int_\Omega |Du|^2 + H(\det Du)\, dx$$

is thus a challenging problem in nonlinear elasticity. The difficulties of this problem stem from the fact that many desirable properties of real deformations, such as $\det Du > 0$ a.e. or condition INV (first introduced by Müller and Spector), are generally not preserved by the weak $W^{1,2}$ limit. The formation of cavities is another possible obstruction to the lower semicontinuity of E, so it is natural to seek a minimizer of E in a subclass that prohibits cavitation. In our recent work, based on the techniques developed by Henao and Mora-Corral, we prove that E is lower semicontinuous on the weak closure of some subclasses of $W^{1,2}(\Omega;\mathbb{R}^3)$, which includes a class $\mathcal{A}_{hom}$ whose members are orientation-preserving $W^{1,2}$ homeomorphisms satisfying Lusin's condition (N). This implies the existence of a minimizer of E in $\mathcal{A}_{hom}$, which is first shown by Doležalová, Hencl and Molchanova. Our existence result holds under a less restrictive coercivity condition for H and, moreover, if we keep their coercivity condition, we show further that the inverse of the minimizer has the Sobolev $W^{1,1}$ regularity.

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