COX RINGS

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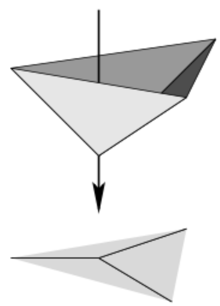
Abstract. The construction of the projective space $\mathbb{P}^n$, as a quotient of $\mathbb{C}^{n+1} \setminus \{0\}$ by the action of $\mathbb{C}^*$, can be described in combinatorial terms in the language of toric varieties, as shown by the following picture for $n = 2$:



The above construction is encoded in the polynomial ring $\mathcal{R}(\mathbb{P}^n) := \mathbb{C}[x_1, \dots, x_{n+1}]$, graded by the integers. More generally one can introduce a wide class of algebraic varieties, called Mori dream spaces [4], and associate to any such variety X a finitely generated graded (by an abelian group) algebra $\mathcal{R}(X)$ called the *Cox ring* [1] of X. As in the case of $\mathbb{P}^n$ the variety X can be reconstructed from $\mathcal{R}(X)$ and an ideal which is defined in a purely combinatorial way. The aim of this talk is to introduce Cox rings and to explain how they behave under birational morphisms $X \to Y$ of Mori dream spaces [2, 3].

References

[1] Ivan Arzhantsev, Ulrich Derenthal, Jürgen Hausen, and Antonio Laface, *Cox rings*, Cambridge Studies in Advanced Mathematics, vol. 144, Cambridge University Press, Cambridge, 2014.

[2] Ulrich Derenthal, Jürgen Hausen, Armand Heim, Simon Keicher, and Antonio Laface, *Cox rings of cubic surfaces and Fano threefolds* (2014), available at arXiv:1409.0799.

[3] Jürgen Hausen, Simon Keicher, and Antonio Laface, *Computing Cox rings*, to appear in Mathematics of Computation (2015), available at arXiv:1305.4343.

[4] Yi Hu and Sean Keel, *Mori dream spaces and GIT*, Michigan Math. J. **48** (2000), 331–348. Dedicated to William Fulton on the occasion of his 60th birthday.