Pontificia Universidad Católica de Chile
Facultad de Matemáticas

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BOOK OF ABSTRACTS

Semiclassical resolvent estimates and resonances free regions for Schrödinger operators with matrix-valued potentials

Marouane Assal
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For semiclassical Schrödinger operators with scalar long-range potentials, a classical result due to Burq ensures that the resolvent norm grows exponentially in the inverse of the semiclassical parameter, and grows linearly near infinity. This result holds without any assumption on the classical dynamics and it implies in particular the absence of resonances exponentially close to the real axis. In this talk, I will present a generalization of these results for semiclassical Schrödinger operators with matrix-valued long-range potentials without any assumption on the eigenvalues crossings. In particular, I will focus on an elementary approach introduced by Datchev based on an explicit global Carleman estimate. If time is enough, I will also present an application of these results to the study of the scattering amplitude.
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The Landau Hamiltonian with a delta-potential supported on curves

Jussi Behrndt
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We investigate the spectral properties of the singularly perturbed self-adjoint Landau Hamiltonian $(i\nabla + A)^2 + \alpha\delta$ acting in the Hilbert space $L^2(\mathbb{R}^2)$ with a δ-potential supported on a finite $C^{2,1}$-smooth curve Σ; here $\alpha \in L^\infty(\Sigma)$ is a position-dependent real coefficient modeling the strength of the singular interaction on Σ.
The talk is based on joint work with P. Exner, M. Holzmann, and V. Lotoreichik.
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Spectral properties of magnetic Hamiltonians on the half-plane

Vincent Bruneau
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We are interested in the Schrödinger operator with constant magnetic field in dimension 2 perturbed by an electric potential. After a review of the results in the whole plane and on a band, we will study the phenomena of spectral accumulation in the case of the half-plane with Dirichlet or Neumann boundary condition. These results on the half-plane, are drawn from works in collaboration with P. Miranda and G. Raikov.
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Spectral asymptotics for pseudodifferential perturbations of the Landau Hamiltonian

Esteban Cárdenas
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In this talk we consider the operator $H_0 + V$, where H_0 is the Landau Hamiltonian and V is a compact pseudodifferential operator. We analyze the asymptotic behavior of the spectrum near the Landau levels and its relation to the symbol of V, which we allow to be of two types. First, we let V have a definite sign and a fast-decaying anti-Wick symbol and, secondly, we consider the case where V has a Weyl symbol with power-like decay. The talk is based on a joint work with Georgi Raikov.
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Spectral continuity for aperiodic quantum systems

Giuseppe De Nittis
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How does the spectrum of a Schrödinger operator vary if the corresponding geometry and dynamics change? Is it possible to define approximations of the spectrum of such operators by defining approximations of the underlying structures? In this talk a positive answer is provided using the rather general setting of groupoid C*-algebras. A characterization of the convergence of the spectra by the convergence of the underlying structures is proved.
In order to do so, the concept of continuous field of groupoids is used. The approximation scheme is expressed through the tautological groupoid, which provides a sort of universal model for fields of groupoids. The use of the Hausdorff topology turns out to be fundamental in understanding why and how these approximations work. The construction presented during the talk is adapted to the case of Schrödinger operator with Delone potential (i.e. quasi-crystals).
The talk is based on a joint work with: S. Beckus and J. Bellissard.
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Transport and thick point spectrum

Cesar R. de Oliveira
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It is shown that a (bounded) pure point quantum Hamiltonian whose spectrum is dense in an interval presents quasiballistic dynamics for a generic set of initial conditions.
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Inverse spectral problems for mass spring systems

Rafael del Rio
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This talk is about a Borg type inverse spectral problem for vibrating linear systems of point masses connected by springs. From the natural frequencies of vibration of the original system and a perturbation of it, we show how the masses and elastic coefficients of the springs can be reconstructed. To accomplish this, rank three perturbations of Jacobi matrices are considered and their associated Green's functions explicitly described in terms of spectral data. We give necessary and sufficient conditions for two given sets of points to be eigenvalues (natural frequencies) of the original and modified system respectively. This is joint work with Luis Silva and Mikhail Kudryavtsev.

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Resonances for a system of Schrödinger operators above an energy level crossing

Setsuro Fujiié
Ritsumeikan University, Japan

We consider a one-dimensional 2×2 matrix-valued self-adjoint operator with two Schrödinger operators $P_j = -h^2\Delta + V_j(x)$, $j = 1, 2$ on the diagonal entries, and small interactions of order h on the anti-diagonal entries. Here h is a semiclassical parameter. The aim is to study the asymptotic distribution of resonances of such an operator as $h \to 0$ in a complex neighborhood of a real fixed energy E_0 under the following setting:

- V_1 has a simple well so that P_1 has eigenvalues near E_0 subject to the Bohr-Sommerfeld quantization rule,
- E_0 is a non-trapping energy for the classical Hamiltonian $\xi^2 + V_2(x)$ corresponding to P_2,
- V_1 and V_2 cross transversally at one point and the interaction is elliptic at this point.

It is known that, near E_0, there exist resonances close to the eigenvalues of P_1. In particular, the imaginary part of resonances is exponentially small when E_0 is below the crossing level (Ashida 2017), and of order $h^{5/3}$ in a h-dependent small neighborhood of E_0 (F-Martinez-Watanabe 2016, 2017).

In this talk, we show that it is of order h^2 above the crossing energy. The key point of the method consists in the microlocal study of solutions at the crossing points in the phase space of the two bicharacteristic curves of P_1 and P_2.

This is a joint work with Andre Martinez (Bologna) and Takuya Watanabe (Ritsumeikan).

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Continuity of the density of states with respect to the probability measure for random Schrödinger operators

Peter D. Hislop
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The main result described in this talk is that the the density of states measure (DOSm) for random Schrödinger operators on $\mathbb{Z}^d$ or on $\mathbb{R}^d$ is weak-* Hölder-continuous in the single-site probability measure. The framework is general enough to extend to a wide range of discrete, random operators, including the Anderson model on the Bethe lattice, as well as random Schrödinger operators on the strip. An immediate application of the main result provides quantitative continuity estimates for the disorder dependence of the DOSm and the integrated density of states (IDS) in the weak disorder regime. These results hold for a general compactly supported single-site probability measure, without any further assumptions. For lat- tice models, the quantitative continuity results hold for non-compactly sup- ported single-site probability measures with finite first moments. This is joint work with C. A. Marx.
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Semiclassical inverse problems for elastic surface waves in isotropic media

Alexei Iantchenko
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We carry out a semiclassical analysis of surface waves in Earth which is stratified near its boundary at some scale comparable to the wave length.
Propagation of such waves is governed by effective Hamiltonians which are non-homogeneous principal symbols of some pseudodifferential operators. Each Hamiltonian is identified with an eigenvalue in the discreet spectrum of a locally 1D Schrödinger-like operator on the one hand, and generates a flow identified with surface wave bicharacteristics in the two-dimensional boundary on the other hand.
The eigenvalues exist under certain assumptions reflecting that wave speeds near the boundary are smaller than in the deep interior. This assumption is naturally satisfied by the structure of Earth's crust and mantle.
Using these Hamiltonians, we obtain pseudodifferential surface wave equations. In case of isotropic medium the equations decouple into Rayleigh and Love waves. In both cases we perform a comprehensive analysis of the recovery of the S-wavespeed from the semiclassical spectrum.
The approach follows the ideas of Colin de Verdière on acoustic surface waves and is joint work with Maarten V. de Hoop, Jian Zhai, Rice University, and Gen Nakamura, Hokkaido University.
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Propagation property and inverse scattering for fractional powers of negative Laplacian

Atsuhide Ishida
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Enss (1983) proved a propagation estimate for the usual free Schrödinger operator that turned out to be very useful for inverse scattering by Enss-Weder (1995). Since then, this method has been called the Enss-Weder time-dependent method. In this talk, first, we introduce the same type propagation estimate for the fractional powers of negative Laplacian. Second, we report about the inverse scattering by applying the Enss-Weder time-dependent method. We find that the high velocity limit of the scattering operator uniquely determines the short-range interactions.
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Zeroth order conjugate operator in N-body Schrödinger operators

Kenichi Ito
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We develop a scheme of proofs for spectral theory of the N-body Schrödinger operators, reproducing and extending a series of sharp results under minimum conditions. The main results are Rellich's theorem and the limiting absorption principle. We present a new proof of Rellich's theorem which is unified with exponential decay estimates studied previously only for L^2-eigenfunctions. Each pair-potential is a sum of a long-range term with first order derivatives, a short-range term without derivatives and a singular term of operator- or form-bounded type. The setup can also include hard-core interactions. Our proof consists of a systematic use of commutators with 'zeroth order' operator, not like the standard 'first order' conjugate operator in the Mourre theory. In particular, our proofs do not rely on Mourre's differential inequality technique. This talk is based on a recent joint work with T. Adachi, K. Itakura and E. Skibsted.
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Spectral shift function for massless Dirac operator

Galina Levitina
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We prove a representation of spectral shift function via the regularised determinant, which is similar to the classical representation of the spectral shift function due to Krein. As an application, we prove continuity result for the spectral shift function for the pair of free massless Dirac operator and its perturbation by electromagnetic potential in dimensions two and three.
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Spectral asymptotics for the Iwatsuka Hamiltonian

Pablo Miranda
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In this talk, we consider the two-dimensional operator $H = H_0 + V$, where H_0 is the Iwatsuka Hamiltonian, that is the Schrödinger operator with a magnetic field that is invariant in one of the spatial variables, and V is a decaying electric potential. We will describe the accumulation rate of the discrete eigenvalues of H in the gaps of its essential spectrum. Afterward, we consider an extension of this problem to the continuous spectrum of H, by the study of the Spectral Shift Function for the pair (H, H_0). To obtain these results it is necessary first to study the band functions of the Iwatsuka Hamiltonian. Then, we will start by showing the asymptotic behavior of the band functions and their dependence on the magnetic field.
This is part of joint works with Nicolas Popoff.
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An enhanced area law for the entanglement entropy in the random dimer model

Peter Müller
LMU Munich, Germany

We consider the random dimer model in one space dimension with Bernoulli disorder. For sufficiently small disorder, we show that the entanglement entropy exhibits (at least) a logarithmically enhanced area law if the Fermi energy coincides with a critical energy of the model where the localisation length diverges.
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Properties of scattering matrix with long-range perturbations

Shu Nakamura
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We consider scattering theory for Schrödinger-type operators with long-range perturbations. We show the (modified) scattering matrix is a Fourier integral operator with the phase function corresponding to the (modified) classical scattering map.
We also show the spectrum of the scattering matrix can be dense point and absolutely continuous, whereas for the short-range case it is well-known that the spectrum is always discrete with the essential spectrum only at 1.
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Spectrum of the Robin Laplacian with singular boundary conditions

Nicolas Popoff
University of Bordeaux, France

In this talk, I will review various results about the spectrum of the Robin Laplacian with "attractive" boundary condition in a bounded domain. As the Robin parameter gets large, the first eigenvalues go to infinity. I will describe the mechanism giving the first orders of the asymptotics when the domain has corners, and give improvements when the domain is regular. This includes the description of the bottom of the spectrum in infinite cones. For 3d cones with regular cross section, we are able to count the number of discrete eigenvalues. Finally, I will consider the case of a variable, vanishing, Robin coefficient. In that case the operator may not be self-adjoint and we are able to compute the indices of deficiency. These results are joint works with Vincent Bruneau, Konstantin Pankrashkin and Sergei Nazarov.
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Index theorems for Fredholm, semi-Fredholm and almost-periodic operators; all in one example

Serge Richard
University of Nagoya, Japan

Based on operators borrowed from scattering theory, several concrete realizations of index theorems will be presented. The corresponding operators belong to some C*-algebras of pseudo-differential operators with coefficients which either have limits at + or - infinity, or which are periodic or asymptotically periodic, or which are uniformly almost periodic. These various situations can be deduced from a single partial isometry which depends on several parameters. From a different point of view, these investigations correspond to a Levinson's type theorem when an infinite number of bound states is involved.
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Random Schrödinger operators arising in aperiodic media

Constanza Rojas-Molina
University of Düsseldorf, Germany

In this talk we discuss how random Schrödinger operators appear naturally as auxiliary models to study Anderson localization in aperiodic media. We work within the framework of Delone operators, which we associate to Anderson-type operators with Bernoulli random variables, and discuss the proof of localization, and its consequences on the spectral type of Delone operators near the bottom of the spectrum. This is joint work with Peter Müller (Munich).
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Spectral properties for non-selfadjoint discrete Schrödinger operators

Diomba Sambou
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Let H_0 be the one-dimensional Schrödinger operator acting on $\ell^2(\mathbb{Z})$, and V be a compact non-selfadjoint perturbation. We relate the regularity properties of V to various spectral properties of the perturbed operator $H_0 + V$. In particular, the structure of the discrete spectrum and the embedded eigenvalues are analysed. Our results are based on a suitable combination of complex scaling techniques and resonance theory. This is a joint work with Olivier Bourget and Amal Taarabt.
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On the asymptotic dynamics of 2-D magnetic quantum systems

Edgardo Stockmeyer
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In this talk I will present results concerning the long time localisation in space (dynamical localisation) of certain two-dimensional magnetic quantum systems. The underlying Hamiltonian may have the form $H = H_0 + W$, where H_0 is rotationally symmetric and has dense point spectrum and W is a perturbation that breaks the symmetry.
Joint work with Esteban Cárdenas, Dirk Hundertmark, and Semjon Wugalter.
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Spectral uncertainty principles at low energy: unique continuation when you don't expect it

Peter Stollmann
TU Chemnitz, Germany

We review an abstract and simple uncertainty principle from 'A. Boutet de Monvel, D. Lenz and P. Stollmann: An uncertainty principle, Wegner estimates and localization near fluctuation boundaries. Math. Z. 2010' and its application to graphs and divergence form operators with rough coefficients based on 'D. Lenz, P. Stollmann and Gunter Stolz: An uncertainty principle and lower bounds for the Dirichlet Laplacian on graphs. arXiv: 1606.07476' and 'P. Stollmann and Gunter Stolz: Lower bounds for Dirichlet Laplacians and uncertainty principles, arXiv:1808.04202'.
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Fredholm module for electromagnetic massive Dirac operator

Fedor Sukochev
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Let D_m, $m > 0$, be a massive Dirac operator on $\mathbb{R}^d$, $d \geq 2$ and let V be an electromagnetic potential. We prove a necessary and sufficient condition for the quantised derivative $i[sgn(D_m + V), 1 \otimes M_f]$ of a bounded function f on $\mathbb{R}^d$ to belong to the weak Schatten ideal $\mathcal{L}_{d,\infty}$. Joint work with G. Levitina and D. Zanin.
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Dirac operatos with decaying disorder

Amal Taarabt
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We will examine transition zones for a class of random Dirac operators with decaying disorder strength.
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A critical Poincaré-Sobolev inequality

Hanne Van Den Bosch
Universidad de Chile

We study a specific Poincaré-Sobolev inequality in bounded domains, that has recently been used to prove a semi-classical bound on the kinetic energy of fermionic many-body states. The corresponding inequality in the entire space is precisely scale invariant and this gives rise to an interesting phenomenon. Optimizers exist for some (most ?) domains and do not exist for some other domains, at least for the isosceles triangle in two dimensions.
In this talk, I will discuss bounds on the constant in the inequality and the proofs of existence and non-existence.
This is joint work with Rafael Benguria and Cristóbal Vallejos (PUC, Chile).
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Scale free unique continuation estimates with three applications

Ivan Veselic
TU Dortmund, Germany

I will present scale free unique continuation estimates for functions in the range of any compact spectral interval of a Schroedinger operator on generalized parallelepipeds. The latter could be cubes, halfspaces, octants, strips, slabs or the whole space. The sampling set is equidistributed. The unique continuation estimates are very precise with respect to the energy, the potential, the coarsenes scale, the radius defining the equidistributed set and actually optimal in some of these parameters. Such quantitative unique continuation estimates are sometimes called uncertainty relations or spectral inequalities, in particular in the control theory community.
These estimates have range of applications. I will present three. The first concerns lifting of edges of components of the essential spectrum, the second Wegner estimates for a variety of random potentials, and the last one control theory of the heat equation.
The talk is based on joint works with Nakic, Taeufer and Tautenhahn, and loosely related with works with Egidi and Seelmann.
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On semiclassical eigenvalue distribution theorems for perturbations of the Landau problem

Carlos Villegas-Blas
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We study two different ways to consider the semiclassical limit of the eigenvalue distribution in clusters around the Landau levels associated to suitable perturbations of the Landau Hamiltonian. In the first one, we consider perturbations of the Landau problem keeping both the magnetic field strength and the value of the Planck's parameter fixed and then study the high energy asymptotics (joint work with A. Pushnitski and G. Raikov). We obtain that such distribution is determined by averages of the perturbation along straight lines on the motion plane. In the second one, we take both the magnetic field strength and the Planck's parameter depending on a parameter in such a way that the eigenvalue distribution is now determined by averages of the perturbation along classical orbits with fixed classical energy (joint work with G. Hernandez Dueñas, S. Perez-Esteva and A. Uribe).
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