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Spectral Theory and Mathematical Physics
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Titles and Abstracts

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Pontificia Universidad Católica de Chile Facultad de Matemáticas

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Mini-courses

Levinson's theorem: an index theorem in scattering theory

SERGE RICHARD

Nagoya University, Japan

Abstract: During this series of lectures, we shall review some recent investigations on the nature of Levinson's theorem. Indeed, this relation between spectral and scattering theory can be recast in a C^* -algebraic framework, and turns out to be an index theorem. For a good understanding of this statement, we shall recall the historical background, introduce the necessary material, and illustrate our purpose with several examples. We shall also present some possible generalizations of Levinson's theorem, and discuss the assets and the weaknesses of the algebraic approach.

Spectral shift function and Witten index

FEDOR SUKOCHEV

School of Mathematics & Statistics, University of NSW, Australia

Abstract: In this series of talks we discuss the notion of the spectral shift function of two self-adjoint operators and its connection with the Witten index.

We start by discussing the classical definitions of the spectral shift function $\zeta(\cdot, H_1, H_2)$ under different assumption on the operators H_1 and H_2 in a Hilbert space and some properties of the function ζ . We shall also review a new approach [4] to the definition of $\zeta(\cdot, H_1, H_2)$ and Krein's Trace Theorem which does not use methods drawn from complex analysis. This approach holds for general σ -finite von Neumann algebras \mathcal{M} of type II and unbounded perturbations from the predual of \mathcal{M} .

Next, we present a connection between the theory of spectral shift function and index theory for certain model operators, in particular we discuss connection with the notion of Witten index, which generalises the notion of the Fredholm index. Our approach is based on the study of the model operator $D_A = (d/dt) + A$ in $L^2(\mathbb{R}; \mathcal{H})$ associated with the operator path $\{A(t)\}_{t=-\infty}^{\infty}$, where $(Af)(t) = A(t)f(t)$ for a.e. $t \in \mathbb{R}$, and appropriate $f \in L^2(\mathbb{R}; \mathcal{H})$ (with \mathcal{H} being a separable, complex Hilbert space). Our setup permits $A(t)$ in \mathcal{H} to be an unbounded relatively trace class perturbation of the unbounded self-adjoint operator A_- , and no discrete spectrum assumptions are made on the asymptotes A_{\pm} .

In the special case, when there is a spectral gap for the operators A_{\pm} at zero, we show that the operator D_A is Fredholm and the Fredholm index can be computed as follows [3]

$$\text{index}(D_A) = \zeta(0_+; D_A D_A^*, D_A^* D_A) = \zeta(0; A_+, A_-).$$

When $0 \in \sigma(A_+)$ (or $0 \in \sigma(A_-)$), the operator D_A ceases to be Fredholm. However, under additional assumption that 0 is a right and a left Lebesgue point of

$\tilde{\zeta}(\cdot; A_+, A_-)$, we prove that 0 is also a right Lebesgue point of $\tilde{\zeta}(\cdot; D_A D_A^*, D_A^* D_A)$ and for the resolvent (respectively, semigroup) regularised Witten index $W_r(D_A)$ (respectively, $W_s(D_A)$) we have that [1]

$$\begin{aligned} W_r(D_A) &= W_s(D_A) = \tilde{\zeta}(0_+; D_A D_A^*, D_A^* D_A) \\ &= [\tilde{\zeta}(0_+; A_+, A_-) + \tilde{\zeta}(0_-; A_+, A_-)]/2. \end{aligned}$$

We also study a special example, when the perturbation of the unbounded self-adjoint operator A_- is not assumed to be relative trace-class. In this example $A_- = \frac{d}{i dx}$ is the differentiation operator on $L^2(\mathbb{R})$ and the perturbation is given by the multiplication operator by a (bounded) real-valued function f on \mathbb{R} . Under mild assumptions on f , we show [2] that

$$\begin{aligned} W_r(D_A) &= W_s(D_A) = \tilde{\zeta}(0_+; D_A D_A^*, D_A^* D_A) \\ &= \tilde{\zeta}(0; A_+, A_-) = \frac{1}{2\pi} \int_{\mathbb{R}} f(s) ds. \end{aligned}$$

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Invited Talks

Spectral properties near thresholds for a model of decay of weak bosons

JEAN-MARIE BARBAROUX

Centre de Physique Théorique, Luminy, France

Abstract: We study spectral properties of a Hamiltonian describing the weak decay of a Z^0 boson into a pair of electron and positron. The operator acts on tensor product of one bosonic and two fermionic Fock spaces.

Our main result states that the nature of the spectrum near thresholds is purely absolutely continuous. As usual for energies near thresholds, the derivation of positive commutator estimates fails with “standard” conjugate operators. Therefore, to derive a Mourre inequality, we follow the strategy of Huebner and Spohn they use for the spectral study of a spin-boson model, and we apply singular Mourre theory with a non self-adjoint conjugate operator.

This is joint work with Jérémy Faupin and Jean-Claude Guillot.

Ground state energy of the magnetic Laplacian on corner domains

VIRGINIE BONNAILLIE-NOËL

CNRS, France

Abstract: The asymptotic behavior of the first eigenvalues of magnetic Laplacian operators with large magnetic fields and Neumann realization in smooth three-dimensional domains is characterized by model problems inside the domain or on its boundary. In two-dimensional polygonal domains, a new set of model problems on sectors has to be taken into account. In this talk, we consider the class of general corner domains. In dimension 3, they include as particular cases polyhedra and axisymmetric cones. We attach model problems not only to each point of the closure of the domain, but also to a hierarchy of “tangent substructures” associated with singular chains. We investigate properties of these model problems, namely continuity, semi-continuity, existence of generalized eigenfunctions satisfying exponential decay. We prove estimates for the remainders of our asymptotic formula. Lower bounds are obtained with the help of a classical IMS partition based on adequate two-scale coverings of the corner domain, whereas upper bounds are established by a novel construction of quasimodes, qualified as sitting or sliding according to spectral properties of local model problems. A part of our analysis extends to any dimension.

This is a joint work with M. Dauge and N. Popoff.

Magnetic resonances for exterior problems

VINCENT BRUNEAU

Institut de Mathématiques de Bordeaux, France

Abstract: We consider $H_0 = (i\nabla - A)^2$ the 3D Schrödinger operator with constant magnetic field. Let K be a simply connected compact domain with smooth boundary and its complementary $\Omega := \mathbb{R}^3 \setminus K$. We study the resonances of the Dirichlet (resp. Neumann, resp. Robin) realization of $(i\nabla - A)^2$ on Ω . We establish the existence of resonance free sectors near Landau level and study a resonance counting function. Consequently we obtain the accumulation of resonances at the Landau levels and in some cases the discreteness of the embedded eigenvalues.

It is a joint work with D. Sambou.

On the Peierls substitution at weak magnetic fields

HORIA CORNEAN

Aalborg University, Denmark

Abstract: Consider an isolated spectral band of a real periodic Bloch Hamiltonian in either two or three dimensions. Assume that the system is weakly perturbed by a smooth and bounded magnetic field. We show that the perturbed spectral projector admits an exponentially localized orthonormal basis and we construct a magnetic matrix which is unitary equivalent with the perturbed band Hamiltonian. If the magnetic field is constant, our magnetic matrix is Hofstadter-like. This is joint work with Ira Herbst (Virginia) and G. Nenciu (Bucharest).

Spectral measures of Jacobi operators with random potentials

RAFAEL DEL RIO

UNAM, Mexico

Abstract: Let H_ω be a self-adjoint Jacobi operator with a potential sequence $\{\omega(n)\}_n$ of independently distributed random variables with continuous probability distributions and let μ_ϕ^ω be the corresponding spectral measure generated by H_ω and the vector ϕ . We consider sets $\mathcal{A}(\omega)$ which depend on ω but are independent of two consecutive given entries of the sequence ω , and prove that $\mu_\phi^\omega(\mathcal{A}(\omega)) = 0$ for almost every ω . This result is applied to show equivalence relations between spectral measures for random Jacobi matrices and to study the interplay of the eigenvalues of these matrices and their submatrices. This is based on joint work with Luis Silva.

Integrable boundary interactions for Ruijsenaars' difference Toda chain

ERDAL EMSIZ

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Abstract: We endow Ruijsenaars' open difference Toda chain with a one-sided boundary interaction of Askey-Wilson type and diagonalize the quantum Hamiltonian by means of deformed hyperoctahedral q -Whittaker functions that arise as a $t = 0$ degeneration of the 6-parameter Macdonald-Koornwinder multivariate Askey-Wilson polynomials. This immediately entails the quantum integrability, the bispectral dual system, and the n -particle scattering operator for the chain in question.

This is joint work with Jan Felipe van Diejen.

Systems of Schrödinger Equations

CLAUDIO FERNÁNDEZ

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Abstract: We review Lavine's energy width and apply it to the study of resonant behavior of time dependent quantum systems. The concept of energy width can be used to estimate the spectral concentration of a quantum state. Also, combined with Mourre estimates, it can be used to obtain a simple way of estimating the sojourn time, which works even in with time dependant Hamiltonians. Concrete examples will be discussed. Above is a joint work with J.Asch (Marseille) and O.Bourget and V. Cortés (PUC).

Boundary values of resolvents of selfadjoint operators in Krein spaces and applications to the Klein-Gordon equation

DIETRICH HÄFNER

University of Grenoble, France

Abstract: In this talk we will present joint work with Vladimir Georgescu and Christian Gérard concerning resolvents of selfadjoint operators in Krein spaces. A Krein space is a Hilbertizable Banach space equipped with a non-degenerate hermitian form, called a Krein scalar product. This Krein scalar product is in general not positive. We obtain a generalization of Mourre's theorem to the Krein space setting: a positive commutator estimate entails a limiting absorption principle. We will discuss in detail the applications to the Klein-Gordon equation coupled to an electromagnetic field. In this case the conserved energy is not positive if the electric field is too strong, but the energy defines nevertheless a Krein scalar product.

On the semicircle law for certain matrix ensembles with dependent entries

WERNER KIRSCH

University of Hagen, Germany

Abstract: We study ensembles of random symmetric matrices whose entries exhibit certain correlations. Examples are distributions of Curie-Weiss-type. We provide a criterion on the correlations ensuring the validity of Wigner's semicircle law for the eigenvalue distribution measure. In case of Curie-Weiss distributions this criterion applies above the critical temperature (i.e. $\beta < 1$). We also investigate the largest eigenvalue of certain ensembles of Curie-Weiss type and find a transition in its behavior at the critical temperature.

This is a joint work with Winfried Hochstättler and Simone Warzel.

Time decay of the wave functions for two-dimensional magnetic Schroedinger operators

HYNEK KOVAŘÍK

University of Brescia, Italy

Abstract: We consider two-dimensional Schroedinger operators with magnetic field. Under certain regularity and decay assumptions on the magnetic and electric field we show that the behaviour of the corresponding resolvent at the threshold and the behaviour of the unitary group for large times are determined by the total flux of the associated magnetic field.

The effective dynamics in curved quantum waveguides and when it does not work

DAVID KREJČIŘÍK

Nuclear Physics Institute ASCR, Czech Republic

Abstract: The Dirichlet Laplacian in a curved three-dimensional tube built along a spatial (bounded or unbounded) curve is investigated in the limit when the uniform cross-section diminishes. Both deformations due to bending and twisting are considered. We show that the Laplacian converges in a norm resolvent sense to the well known one-dimensional Schroedinger operator whose potential is expressed in terms of the curvature of the reference curve, the twisting angle and a constant measuring the asymmetry of the cross-section.

Contrary to previous results, we allow reference curves to have non-continuous and

possibly vanishing curvature. For such curves, the distinguished Frenet frame need not exist and, moreover, the known approaches to establish the result do not work. We ask the question under which minimal regularity assumptions the effective one-dimensional approximation holds.

Our main ideas how to establish the norm-resolvent convergence under the minimal regularity assumptions are to use an alternative frame defined by a parallel transport along the curve and a refined smoothing of the curvature via the Steklov approximation. On the negative side, we construct an explicit waveguide for which the usefulness of the spectral information provided by the effective Hamiltonian is rather doubtful.

Fractional Laplacians – Navier vs Dirichlet

ALEXANDER NAZAROV¹

St. Petersburg Department of the Steklov Institute
St. Petersburg State University

Abstract: Let Ω be a bounded domain with smooth boundary. We compare two natural types of fractional Laplacians $(-\Delta)^s$, namely, the “Navier” and the “Dirichlet” ones. We denote their quadratic forms by $Q_{s,\Omega}^N$ and $Q_{s,\Omega}^D$, respectively.

Theorem 1. *Let $s > -1$, $s \notin \mathbb{N}_0$. Then for $u \in \text{Dom}(Q_{s,\Omega}^D)$, $u \neq 0$, the following relations hold:*

$$\begin{aligned} Q_{s,\Omega}^N[u] &> Q_{s,\Omega}^D[u], \quad \text{if } 2k < s < 2k + 1, \quad k \in \mathbb{N}_0; \\ Q_{s,\Omega}^N[u] &< Q_{s,\Omega}^D[u], \quad \text{if } 2k - 1 < s < 2k, \quad k \in \mathbb{N}_0. \end{aligned}$$

Moreover, for $u \in \text{Dom}(Q_{s,\Omega}^D)$ the following facts hold (here $F(\Omega)$ stands for the class of smooth and bounded domains containing Ω).

$$\begin{aligned} Q_{s,\Omega}^D[u] &= \inf_{\Omega' \in F(\Omega)} Q_{s,\Omega'}^N[u], \quad \text{if } 2k < s < 2k + 1, \quad k \in \mathbb{N}_0; \\ Q_{s,\Omega}^D[u] &= \sup_{\Omega' \in F(\Omega)} Q_{s,\Omega'}^N[u], \quad \text{if } 2k - 1 < s < 2k, \quad k \in \mathbb{N}_0. \end{aligned}$$

Theorem 2. *Let $0 < |s| < 1$, and let $f \in \text{Dom}(Q_{s,\Omega}^D)$, $f \geq 0$, $f \neq 0$. Then the following relations hold:*

$$\begin{aligned} (-\Delta_\Omega)_N^s f &> (-\Delta_\Omega)_D^s f, \quad \text{if } 0 < s < 1; \\ (-\Delta_\Omega)_N^s f &< (-\Delta_\Omega)_D^s f, \quad \text{if } -1 < s < 0. \end{aligned}$$

Here all inequalities are understood in the sense of distributions.

This talk is based on joint papers with Roberta Musina, see [1], [2].

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Directed entangled states, entropy production and reversibility²

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Abstract: Reversibility of a dynamical system is a subject which has been in discussion since the dawn of thermodynamics. In the first third of the twentieth century, Onsager [8] provided a sound analysis of the concept in physics. The linear response theory summarizes Onsager studies on the relation of reversibility with both, the so called *detailed balance conditions* and the zero *production of entropy*.

In the theory of stochastic processes, reversibility has been widely studied within the framework of Markov chains and processes. This approach includes the statistical physics analysis of classical open systems. A number of authors (see eg. [6], [7]) have been studying a characterization of quantum reversibility via a system of axioms inspired in Onsager reciprocity relations. In [2], Fagnola and I, we introduced a notion of *quantum entropy production* by extending the approach of classical Markov processes and using a notion of *quantum detailed balance*, which was separately studied in all its possible variants by Fagnola and Umanità in [4].

A crucial aspect in all the above studies of reversibility is the concept of *direction of a given evolution*. One indeed implicitly assume a representation of what "forward" or "backward" means, before establishing a corresponding notion of equilibrium and reversibility.

Entanglement will be used in my conference to introduce directions of an evolution described by a Quantum Markov Semigroup (QMS). This idea supports our joint research with Franco Fagnola in [3], as well as that of Agredo [1]. And I will summarize necessary and sufficient conditions for reversibility of an important class of these QMS. This class includes the classical case as well as genuinely non commutative semigroups obtained from weak coupling limits.

²This research has been partially supported by FONDECYT grant#1120063 and the *Stochastic Analysis Research Network*, CONICYT-PIDA grant ACT1112.

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Pointwise estimates on derivatives of Coulombic wavefunctions

THOMAS ØSTERGAARD SØRENSEN

University of Munich, Germany

Abstract: In this talk we prove (optimal) a priori pointwise bounds for all derivatives of non-relativistic Coulombic eigenfunctions ψ , involving negative powers of the distance to the singularities of the many-body potential. If time permits, we discuss consequences for the corresponding electron density.

This is joint work with S. Fournais (Aarhus) and T. Hoffmann-Ostenhof (Vienna).

Nodal sets of thin curved layers

MATEJ TUŠEK

Czech Technical University in Prague

Abstract: The talk is concerned with the location of nodal sets of eigenfunctions of the Dirichlet Laplacian in thin tubular neighbourhoods of hypersurfaces of the Euclidean space of arbitrary dimension. In the limit when the radius of the neighbourhood tends to zero, it is known that spectral properties of the Laplacian are approximated well by an effective Schrödinger operator on the hypersurface with a potential expressed solely in terms of principal curvatures. By applying techniques of elliptic partial differential equations, we strengthen the known perturbation results to get a convergence of eigenfunctions in Hölder spaces. This enables us in particular to conclude that every nodal set has a non-empty intersection with the boundary of the tubular neighbourhood.

The talk is based on a joint work with David Krejčířík.

Quantitative unique continuation estimate and Wegner estimate for the standard random breather potential

IVAN VESELIĆ

Chemnitz Technical University, Germany

Abstract: We present a new scale-free, quantitative unique continuation estimate for Schroedinger operators in multidimensional space. Depending on the context such estimates are sometimes called uncertainty relations, observations inequalities or spectral inequalities. To illustrate its power we prove a Wegner estimate for Schroedinger operators with random breather potentials. Here we encounter a non-linear dependence on the random coupling constants, preventing the use of standard perturbation theory. The proofs rely on an analysis of the level sets of the random potential, and can be extended to a rather general framework.

Student Talks

C*-algebraic covariant structures

HAROLD BUSTOS

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Abstract: We introduce *covariant structures* $\{(\mathcal{A}, \kappa), (a, \alpha), (\tilde{a}, \tilde{\alpha})\}$ formed of a separable C*-algebra \mathcal{A} , a measurable twisted action (a, α) of the second-countable locally compact group G , a measurable twisted action $(\tilde{a}, \tilde{\alpha})$ of another second-countable locally compact group \tilde{G} and a strictly continuous function $\kappa : G \times \tilde{G} \rightarrow UM(\mathcal{A})$ suitably connected with (a, α) and $(\tilde{a}, \tilde{\alpha})$. Natural notions of covariant morphisms and representations are considered, leading to a sort of twisted crossed product construction. Various C*-algebras emerge by a procedure that can be iterated indefinitely and that also yields new pair of twisted actions. Some of these C*-algebras are shown to be isomorphic. The constructions are non-commutative, but are motivated by Abelian Takai duality that they eventually generalize.

Stahl's theorem (aka BMV conjecture): Insights and intuition on its proof

FABIEN CLIVAZ

ETH, Zurich

Abstract: The Bessis-Moussa-Villani conjecture states that the trace of $\exp(A - tB)$ is, as a function of the real variable t , the Laplace transform of a positive measure, where A and B are hermitian matrices. The long standing conjecture was recently proved by Stahl and streamlined by Eremenko. We report on that proof and recall the significance of the result.

Local statistics for some random operators

DHRITI RANJAN DOLAI

Institute of Mathematical Sciences, Chennai, India

Abstract: We present results on local statistics for the Anderson model in two different cases. One case when the potential have singular continuous distribution and the other case when the random potential decays.

Local spectral asymptotics for metric perturbations of the Landau Hamiltonian

TOMÁS LUNGENSTRASS
Facultad de Matemáticas
Pontificia Universidad Católica de Chile

Abstract: We consider metric perturbations of the Landau Hamiltonian. We investigate the asymptotic behavior of the discrete spectrum of the perturbed operator near the Landau levels, for perturbations with power-like decay, exponential decay or compact support. This is joint work with Georgi Raikov.

Pauli operators in dimension two with repulsive potentials

JOSEF MEHRINGER
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Pontificia Universidad Católica de Chile

Abstract: We discuss spectral features of the Pauli operator in \mathbb{R}^2 , describing a spin-1/2 particle in a magnetic field B (perpendicular to the plane), under the influence of a repulsive electric potential V . In particular, we establish criteria for discrete and dense pure point spectrum.

Mourre theory for Schrödinger operators on topological crystals

DANIEL PARRA
Institut Camille Jordan, Lyon, France

Abstract: In this talk we consider Schrödinger operators on topological crystals, considered to be a covering graph X over a finite graph X_0 such that the transformation group is free Abelian. Via Floquet-Bloch decomposition one can determine the spectra of the Laplacian and show that it is absolutely continuous, the main tool being the Mourre theory for analytically fibered operators. After presenting this setting, we will focus our attention on the perturbed operator $\Delta_X + V$ and show that, provided that one assumes a certain decay on V , the perturbation leave essentially stable the spectral structure of X . Several examples of topological crystal will be presented. This is our ongoing doctoral work under the supervision of Serge Richard.

Phase transitions in probabilistic cellular automata : an intermediate model

HANNE VAN DEN BOSCH

Facultad de Física

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Abstract: Probabilistic cellular automata (PCA) are a special kind of Markov chains that are studied in mathematical physics and computer science. In these models both space and time are discretized, which allows for a simple formulation and easy numerical simulation. In spite of this apparent simplicity, PCA feature a wide variety of interesting phenomena. In particular, the competition between random noise and some deterministic transition rule may give rise to two opposed types of long term behavior: ergodicity when all information about the initial condition disappears as time tends to infinity, versus non-ergodicity when the asymptotic state depends on the initial condition. The transition between both regimes is called a dynamical phase transition. The occurring of these transitions has only been proven in a reduced class of PCA, while numerical simulations and mean field models suggest they are common to a much wider class. This talk will give a short introduction to the subject and introduce an intermediate model between a type of PCA and their mean field model for which we were able to prove that a phase transition occurs. (joint work with Jean Bricmont, Université Catholique de Louvain)